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MATDIP401

Fourth Semester B.E. Degree Examination, December 2012

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Prove that the angle between two lines whose direction cosines are (ℓ_1, m_1, n_1) and (ℓ_2, m_2, n_2) is $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$. **(06 Marks)**

b. Find the projection of the line AB on CD where A = (1, 3, 5), B = (6, 4, 3), C = (2, -1, 4) and D = (0, 1, 5). **(07 Marks)**

c. Find the angle between any two diagonals of cube. **(07 Marks)**

2 a. Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to $5x + y + 4z = 0$. **(06 Marks)**

b. Show that the points (0, -1, 0), (2, 1, -1), (1, 1, 1) and (3, 3, 0) are coplanar. **(07 Marks)**

c. Find the equation of the plane through the points (1, 0, -1), (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$. **(07 Marks)**

3 a. Find the value of λ such that the vectors $\lambda i + j + 2k$, $2i - 3j + 4k$ and $i + 2j - k$ are coplanar. **(06 Marks)**

b. If $\vec{a} = 4i + 2j - k$, $\vec{b} = 2i - j$ and $\vec{c} = j - 3k$, find (i) $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c})$, (ii) $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$. **(07 Marks)**

c. Find the cosine and sine of the angle between the vectors $2i - j + 3k$ and $i - 2j + 2k$. **(07 Marks)**

4 a. Find the components of velocity and acceleration at t = 2 on the curve, $\vec{r} = (t^2+1)i + (4t-3)j + (2t^2-6t)k$ in the direction of $i + 2j + 2k$. **(06 Marks)**

b. Find the angle between the tangents to the curve $\vec{r} = \left\{t - \frac{t^3}{3}\right\}i + t^2 j + \left\{t + \frac{t^3}{3}\right\}k$ at $t = \pm 3$. **(07 Marks)**

c. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

5 a. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1). **(06 Marks)**

b. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$. **(07 Marks)**

6 a. Find Laplace transform of $(2t + 3)^2$. **(05 Marks)**

b. Find Laplace transform of $e^{2t}\cos 3t$. **(05 Marks)**

c. Find $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$. **(05 Marks)**

d. Using Laplace transform, evaluate $\int_0^{\infty} e^{-2t} t \cos t \, dt$. **(05 Marks)**

7 a. Find inverse Laplace transform of $\frac{s}{s^2 + 4s + 13}$. **(06 Marks)**

b. Find $L^{-1}\left\{\frac{1}{(s^2 + 3s + 2)(s + 3)}\right\}$. **(07 Marks)**

c. Find $L^{-1}\left\{\log\left(\frac{s^2 + 1}{s^2 + s}\right)\right\}$. **(07 Marks)**

8 a. Solve the differential equation $y'' + 4y' + 3y = e^{-t}$ with $y(0) = 1$ and $y'(0) = 1$ by using Laplace transforms. **(10 Marks)**

b. Solve by using Laplace transforms $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$ with $x = 1$, $y = 0$ at $t = 0$. **(10 Marks)**

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